Student Self-Assessment of Mathematics (SSAM) for Intermediate Algebra

Answer key

1. Find the value of $3x - 4y$ if $x = -2$ and $y = 5$

   To find the value, substitute the given values in for $x$ and $y$

   $3x - 4y$

   Substitute (-2) for $x$ and (5) for $y$

   $3(-2) - 4(5)$

   When you multiply a positive number and a negative number, the answer is always negative

   When you multiply 2 positive numbers or 2 negative numbers, the answer is always positive

   $(-6) - (20)$

   Remember when you subtract numbers, you are really “adding the opposite”

   $(-6) + (-20)$

   $(-6)$ plus “the opposite of” 20

   $(-26)$

   Hint: Visualize a negative sign as backing up. If you back up 6 steps and then back up 20 more steps, you have backed up 26 steps. Caution: This is not quite true as distance is always positive. You cannot really go a negative distance.

2. Last December, the temperature on a certain day rose from (-7) degrees Fahrenheit to 5 degrees above zero Fahrenheit. How much did the temperature rise altogether that day?

   If you cannot solve this problem, try drawing a picture. This asks you to “find the distance between the two temperatures”. Remember, distance is always positive.

   $(5) - (-7)$

   Add the opposite

   $(5) + (7)$

   the opposite of (-7) is (+7)

   12 degrees

   Always label your answer if possible

3. Without a calculator calculate: $24 \div \frac{3}{4} - 2 \cdot 4$

   Order of operations is important. PEMDAS: Parentheses, Exponents, Multiply/Divide (whichever comes first), Add/Subtract (whichever comes first)

   $24 \div \frac{3}{4} - 2 \cdot 4$

   Division first

   $24 \cdot \frac{4}{3} - 2 \cdot 4$

   To divide fractions, you multiply by the reciprocal (copy-dot-flip)
To multiply fractions, multiply straight across the top then straight across the bottom.

\[
\frac{24 - 4}{3} = 2 \cdot 4
\]

Order of operations calls for multiplication before subtraction.

\[
32 - 2 \cdot 4
\]

Subtract (Add the opposite \(32 + (-8)\))

\[
32 - 8
\]

\[
24
\]

4. What is the slope and y-intercept of the graph? Graph the equation.

\[
y = 2x + 3
\]

The equation is in slope-intercept form \(y = mx + b\) where \(m\) is the slope and \(b\) is the y-intercept.

Slope = 2; y-intercept = 3

5. What is the slope of the line that goes through the points (1, 4) and (-1, -2)?

Hint: plot the points and connect the dots then count “rise” over “run” OR use the formula which is also \(\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}\) which stands for \(\frac{\text{the change in y}}{\text{the change in x}}\)

OR the difference between the y-values divided by the difference between the x-values \(\frac{y_2 - y_1}{x_2 - x_1}\)

OR subtract the y-values, subtract the x-values then divide y by x

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{1 - (-1)}
\]

Substitute the x and y values in

\[
\frac{4 + (2)}{1 + (1)} \quad \text{Add the opposite}
\]

\[
\frac{6}{2} = 3
\]

The slope is 3

6. A 42-inch wire is to be cut into two pieces. One piece must be exactly twice as long as the other piece. How long should the shorter piece be?

Hint: Draw a picture.
Let the smallest piece = x

Let the bigger piece be twice as long as x (or 2x)

Together the pieces must be 42 inches.

\[ x + 2x = 42 \]  Set up the equation using what you know

\[ 3x = 42 \]  Combine like terms

\[ \frac{3x}{3} = \frac{42}{3} \]  Get x by itself (divide both sides by 3)

\[ x = 14 \text{ in.} \]  The shorter piece is 14 inches the longer piece is 28

Check: One piece is 14 in. and the other is twice 14 = 28 in. Together 14 + 28 = 42 inches√

7. Solve for x: \( 2(x-3) = 3x + 5 \)

\[ 2(x-3) = 3x + 5 \]  Order of operations: PEMDAS

\[ 2x - 2(3) = 3x + 5 \]  Use the distributive property to remove parentheses

\[ 2x - 2x - 6 = 3x - 2x + 5 \]  Get variables on same side of equal sign (Subtract 2x from each side)

\[ -6 + (-5) = x + 5 - 5 \]  Move constants to same side of equal sign (Add opposite)

\[ (-11) = x \]

Check: \( 2(x-3) = 3x + 5 \)

\[ 2(-11 -3) = 3(-11) + 5 \]

\[ 2(-11 + -3) = -33 + 5 \]

\[ -28 = -28 \]√

8. Multiply: \( 3x^2(5x^3 - 2x + 7) \)

What is the numerical coefficient of the first term? 15
What is the numerical coefficient of the second term? -6
What is the numerical coefficient of the third term? 21
What is the degree of the first term? 5
What is the degree of the second term? 3
What is the degree of the third term? 2

3x^2(5x^3 - 2x + 7) Use the distributive property to remove parentheses
15x^5 - 6x^3 + 21x^2 You add the exponents when multiplying

A numerical coefficient is the number in front of the variable
The degree is the power or exponent

Hint: If you cannot remember if you are adding or multiplying the exponents, write the whole thing out without exponents.
Ex: x^2 = xx and x^3 = xxxx so altogether you have xx · xxx = xxxxx = x^5

9. The length of a rectangular bed is 2 feet less than 2 times its width. Find the length of the bed if the perimeter is 32 feet.

Hint: Draw a picture

\[ L = \text{length} \]
\[ W = \text{width} \]

Let \( w \) = width
Let \( l \) = length

\( l \) is 2 feet less than 2 times \( w \)
\( l = 2w - 2 \)

The perimeter is the distance around the outside so \( P = w + w + l + l \) OR \( P = 2w + 2l \)
\( P = w + w + (2w-2) + (2w-2) \) OR \( P = 2w + 2l \)
32 = w + w + 2w -2 + 2w - 2
32 = 6w - 4
32 + 4 = 6w - 4 + 4
36 = 6w
36/6 = 6w/6
6 = w
L = 2w -2

Check: \( P = 2w + 2L \)
\( P = 2(6) + 2(10) \)
P = 12 + 20
P = 32 ft\( \sqrt{ } \)
10. Simplify the expression: \( \frac{1}{2} \cdot 3 + (-5) \). Leave your answer in fractional form.

To add or subtract fractions, you must have a common denominator.

8 is common to both denominators so change \( \frac{1}{2} \) to \( \frac{4}{8} \) so

\[
\frac{4}{8} - \frac{3}{8} + (-5)
\]

\[
\frac{1}{8} + (-5) \quad \text{change } -5/1 \text{ to } 8^{th}
\]

\[
\frac{1}{8} + (-40/8) \quad \text{and then}
\]

\[
\frac{-39}{8}
\]

11. Solve for B: \( A = BC + D \)

\[
A - D = BC + D - D \quad \text{We are trying to get B by itself: Subtract the D from both sides}
\]

\[
A - D = BC \quad \text{Still trying to get B by itself: divide each side by C}
\]

\[
\frac{A-D}{C} = \frac{BC}{C}
\]

\[
\frac{(A-D)}{C} = B
\]

12. Evaluate the algebraic expression \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) when \( a = 2, b = -3, \) and \( c = -2 \)

\[
\frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)} \quad \text{Substitute all values in and follow order of operations}
\]

\[
\frac{-(-3) \pm \sqrt{9 - (-16)}}{4}
\]
\[
\frac{3\pm\sqrt{9+16}}{4} = \frac{3\pm\sqrt{25}}{4} = \frac{3\pm 5}{4} \quad \text{now there are 2 answers}
\]

\[
\frac{3+5}{4} = \frac{8}{4} = 2 \quad \text{and} \quad \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}
\]

13. If 5 gallons of stain are needed to stain 2 wooden decks, how many gallons are needed for 5 decks?

5 gallons = 2 decks

\[? \text{ gallons} = 5 \text{ decks}\]

This is a proportion problem. Set it up by keeping like quantities in the same places: gallons to gallons and decks to decks

\[
gallons \quad \text{decks}
gallons \quad \text{decks}
\]

\[
\frac{5 \text{ gallons}}{x \text{ gallons}} = \frac{2 \text{ decks}}{5 \text{ decks}}
\]

\[
\frac{5}{x} = \frac{2}{5} \quad \text{Solve by cross multiplying}
\]

5 (5) = x (2)
25 = 2x
25/2 = 2x/2
12.5 = x

12 ½ gallons

14. Simplify: \((5x^3y^4)^2\)

\((5x^3y^4)^2 \text{ means } (5x^3y^4)(5x^3y^4) \text{ now multiply}
5 \cdot 5 \cdot x^{3} \cdot x^{3} \cdot y^{4} \cdot y^{4}
25x^{6}y^{8}\]
15. Everything in the hardware store is on sale for a 25% discount. What would a hammer that was originally priced at $16.95 cost on sale?

Discount means Less or subtract
Find 25% of the original cost and then subtract to find the sale price

$16.95 (25%)
$16.95 (.25) = $4.2375 or $4.24
$16.95 - $ 4.24 = $ 12.71